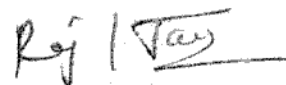# SCHEME OF EXAMINATION AND COURSE OF STUDY

SYLLABUS

FACULTY OF SCIENCE

COURSE WORK FOR M. Phil./ Ph.D. STUDENTS IN MATHEMATICS

EXAMINATION 2020 AND ONWARDS

Dy. Registrar (Academic-I)
University of Rajasthan
Jaipur

Course work for Ph.D. students in Mathematics – 2020 and onwards

Scheme of Examination : Semester-I

For Semester – I teaching is common for both Ph.D. and M.Phil. students. (Papers, Syllabus and Scheme of Examination are also common)

There shall be four papers in all. Two papers are compulsory and two papers are elective.   3 hrs. duration
Each Theory paper and dissertation   Max. Marks – 80
Internal assessment   Max. Marks – 20
                                                               Total Marks – 100
                                                               (for each four papers)

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Semester – I is common for both Ph.D. and M.Phil. students.
Teaching Pattern:
Self study with guidance from a Faculty Member, who will act as Supervisor. To start with, each Supervisor will give one seminar lecture to all the students offering the paper so as to provide a model to the students.

For Semester – I,
The papers are as follows:
Paper – I :   Research Methodology
Paper – II :   Analysis of Published Research Paper
               (A short dissertation is to be submitted by each student under the supervision of a faculty member)

Each student has to opt two papers out of the following
Papers III to IX :

Paper – III :   Advanced Numerical Analysis
Paper – IV :   Generalized Hypergeometric Functions and Fractional Calculus
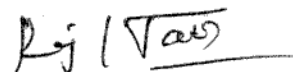Paper – V :   Operations Research-I
Paper – VI :   Mathematical Modelling and Perturbation Methods
Paper – VII :   Integral Transforms and Advanced Tensor Analysis
Paper – VIII : Advanced Graph Theory-I
Paper – IX :   Relativistic Cosmology and Differential Forms

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Note:

1.   There will be four hour teaching in a week per paper including dissertation.

2.   For pass in course work for Ph.D. and M.Phil. course a candidate shall be required to obtain (a) at least 40% marks in each paper separately in internal assessment and external assessment and (b) a minimum of 50% marks in aggregate of all the papers prescribed for the examination (internal and external assessment taken together).

Paper – I : Research Methodology

3 hrs. Duration   Theory Paper   Max.Marks-80
                                 Internal Assessment   Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Four Units. Two questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking one question from each Unit. All questions carry equal marks.

Unit I: Concept of Research, Types and approaches, Ethics in Research, Types and Methods of Research, Report Writing, The First Draft, title page, table of contents, headings and sub-headings, footnotes, tables and figures, appendix, References etc.

Unit II: Plagiarism checking, Editing, Post Writing Stage, Elements of Thesis, Title Page, Table of Contents, Proof Reading, Use of Computer in Research, Use of technology and other equipments in Research.

Unit III: Preparation of Power Point Presentation, Introduction to UGC INFONET, INFLIBNET and ERNET etc. Writing a Research Paper, Choosing a Topic.
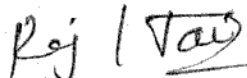
Unit IV: Preparing a Working Bibliography, Use of available Computer Softwares and tools in Research

Paper – II :   Analysis of Published Research Paper

                    (A short dissertation is to be submitted by each student under the supervision of a faculty member)

Dissertation Evaluation (External)   Max.Marks-80
                    Internal Assessment   Max.Marks -20

(Internal Assessment will be done by the supervisor concerned on the basis of the performance and aptitude towards research of the candidate)

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Paper – III: Advanced Numerical Analysis

3 hrs. Duration	**Theory Paper**	**Max.Marks-80**
	Internal Assessment	**Max.Marks-20**

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1: Errors – Errors in Numerical Calculation, Numbers and their accuracy, Errors and their analysis, General error formula, Error in a series approximation. Number representation. Numerical Solutions of Integral Equations – Integral equations, Fredholm integral equation, Finite difference methods, Chebyshev series method, Method using generalized quadrature, Method for degenerate kernels.

Unit 2: Numerical Solution of Ordinary Differential Equations – Two-Point BVPs, Difference methods – Second order, Numerov fourth order methods, Linear ordinary differential equations, Non-linear ordinary differential equations, Non-uniform grid methods for the second order BVP.

Paper – IV : Generalized Hypergeometric Functions and Fractional Calculus

3 hrs. Duration	**Theory Paper**	**Max.Marks-80**
	Internal Assessment	**Max.Marks-20**

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1: Genralized hypergeometric function – Definition, Convergence of the series for $_pF_q$, Differential equation and its solution. Contiguous function relations. Saalschutz's theorem, Whipple's theorem. Dixon's theorem. Contour integral representation for $_pF_q$. Eulerian type integrals involving $_pF_q$. Integral representation for $_pF_q$.

Unit 2: Fractional Calculus – Definition and elementary properties of Riemann-Liouville fractional integrals and derivatives. Derivatives of the fractional integral and the fractional integral of derivatives. Leibniz's formula for fractional integral and fractional derivatives. Law of exponents. Images of elementary and generalized hypergeometric functions under fractional integrals and derivatives.

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Paper-V: Operations Research-I

| 3 hrs. Duration | Theory Paper | Max.Marks-80 |
| | Internal Assessment | Max.Marks-20 |

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1: Theory of Games – Basic definitions, Saddle point, Optimal strategies and the value of game. Fundamental theorem of game theory. 2x2 games without saddle point. Graphical method for 2xn and mx2 games. Inventory Models – Definition, elementary inventory models e.g. EOQ model without and with shortages and EOQ with constraints.

Unit 2: Replacement and Reliability Models – Replacement of items that deteriorate, Replacement of items that fail completely and other replacement problems. Queueing Theory – Definition, Queueing system. Arrival distribution theorem, Distribution of departures, Probabilistic queueing models – Models I to IX; Mixed queueing models – Model X; Deterministic queueing model – model XI.

Paper-VI: Mathematical Modelling and Perturbation Methods

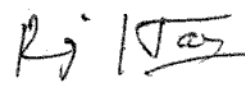| 3 hrs. Duration | Theory Paper | Max.Marks-80 |
| | Internal Assessment | Max.Marks-20 |

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit – I : Perturbation Methods

Regular and singular perturbation: Examples of regular and singular perturbation problems. Method of Multiple Scales : Flow of fluid in a channel of slowly varying cross section, Solution of Mathieu's equation, Solution of the Vander Pal equation by the two-variable method.Lindstedt-Poincare method. Matched Asymptotic Expansions: Prandtl's matching principle deduction of the boundary layer equations for the flow of fluid past a flat plate, improving Stokes' solution of flow past a sphere at low Reynolds number, Whilehead's paradox. Van Dyke's matching principle.

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Unit – II : Mathematical Modelling

Newtonian and non-Newtonian fluids, constitutive equations, some important class of non-Newtonian fluids. Fundamental equations of motion and continuity. Blood rheology, Cardiovascular system. Hagen-Poiseuille flow. Steady laminar flow of blood in a circular tube (Newtonian fluid and Casson fluid). Oscillatory flow of a viscous fluid through an elastic tube. Oscillating flow through a circular tube (Newtonian fluid and two-phase fluid). Oscillatory blood flow. Blood flow through artery with mild stenosis (Newtonian fluid case). Peristaltic flows in a channel and a circular tube.

Paper-VII: Integral Transforms and Advanced Tensor Analysis

| 3 hrs. Duration | Theory Paper | Max.Marks-80 |
| | Internal Assessment | Max.Marks-20 |

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit – I : Integral Transforms

Construction of an integral transform to Strum-Liouville problems. Application of integral transforms namely, Fourier exponential, sine and cosine transforms, Laplace transform, Hankel transform, Mellin transform, Finite Fourier Cosine, Finite Fourier sine and finite Hankel transforms in solving initial value problem and initial boundary value problems for linear differential and integral equations.

Evaluation of integrals by change of order of integration and summation technique, interchange of order of integrations method, and by the application of integral transforms.

Unit – II : Advanced Tensor Analysis

Generalized Kronecker delta, Krutkov tensor, Ricci rotation coefficients and geometrical properties, Hypersurface, Gauss formulae, Curvature of a curve in a hypersurface, normal curvature of a hyper surface, Conformal invariance, classification of gravitational field, Space-matter tensor, Conharmonic curvature tensor, Conharmonically flat space, Symmetry, Maximally symmetric space, spherical, plane and cylindrical symmetries.

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Paper-VIII: Advanced Graph Theory-I

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit-I:

Graphic sequences, The Petersen graph, Cayley's formula, Matrix tree theorem, Matchings, Maximal and maximum matchings, Alternating and augmenting paths, Hall's matching condition, Marriage theorem, Vertex and edge covers, Independent sets, Algorithm for computing maximum bipartite matching, Planar embeddings, Planar graphs, Dual graphs, Outerplanar graphs.

Unit-II:

Line Graphs - Some properties of line graphs, Characterization of line graphs, Special line graphs, Line graphs and traversability. The automorphism group of a graph, Operation on Permutation groups, The group of a composite graph, Graphs with a given group, Symmetric graphs.

Paper- IX: Relativistic Cosmology and Differential Forms

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1:

Lie derivative of a Tensor field, Scalar function, Contravariant and covariant vectors, Covariant tensor of rank two, Symmetry and killing equations, Integrability of killing equation, Geodesic deviation, Conformal curvature tensor, its properties, Algebraic classification of conformal curvature tensor.

Unit 2:

Basic equations of isotropic cosmology, singularity and Singularities in isotropic models, Red Shift in non-static form of de-Sitter universe, Einstein-space, Cosmological principles (perfect, ordinary and weak), Relativistic models not obeying cosmological principle, Godel universe and its properties.

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M.Phil. Mathematics Examination 2020 and onwards
Scheme of Examination : Semesters-II

For Semester – II

3 hrs. Duration	**Theory Paper**	**Max.Marks-80**
	Internal Assessment	**Max.Marks-20**

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

	Dissertation	**Max.Marks-100**

Teaching Pattern:

Self study with guidance from a Faculty Member, who will act as supervisor. To start with, each Supervisor will give one seminar lecture to all the students offering the paper so as to provide a model to the students.

For Semester – II,

The papers are as follows:

Paper – I :   Advanced Mathematical Analysis

Paper – II :   Analysis of Published Research Paper
(A short dissertation is to be submitted by each student under the supervision of a faculty member)

Each student has to opt two papers out of the following Papers III to VIII:

Paper – III :   Advanced Numerical Analysis

Paper – IV :   Generalized Hypergeometric Functions and Fractional Calculus
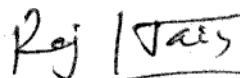
Paper – V :   Operations Research-II

Paper – VI :   Non-Newtonian Fluid Dynamics

Paper – VII :   Advanced Graph Theory-II

Paper – VIII :Relativistic Cosmology and Differential Forms

Note:

1. There will be four hour teaching in a week per paper including dissertation.

2. For pass in course work for Ph.D. and M.Phil. course a candidate shall be required to obtain (a) at least 40% marks in each paper separately in internal assessment and external assessment and (b) a minimum of 50% marks in aggregate of all the papers prescribed for the examination (internal and external assessment taken together).

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Note: Paper I and II are compulsory and two papers out of the papers III to VIII will remain the same which the candidate opted in Semester- I.

Paper – I : Advanced Mathematical Analysis (Compulsory for all candidates)

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Four Units. Two questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking one question from each Unit. All questions carry equal marks.

Unit 1:

Real valued convex function on normed spaces, Bilinear forms, Lax Milligram Lemma, Differentiation and Integration in normed spaces, Gateaux derivative, Frechet derivative. Brouwer's and Schauder's fixed point theorems. Banach Fixed Point Theorem. Application of Banach Contraction Principle to matrix equation.

Unit 2:

Sylow's theorem, Semigroup, Semigroup of relations on a set of Congruences, Factor groupoids and homomorphisms, Homotopy Type, The Fundamental Group, Covering Spaces.

Paper – II : Dissertation :

Each candidate has to submit a dissertation, based upon the papers opted by the candidate in M.Sc./M.Phil. Course, equal to a paper carrying 100 marks.
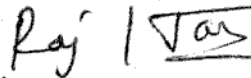
External Evaluation	Max.Marks-100

Paper – III: Advanced Numerical Analysis

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

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Unit 1:

Numerical Solution of Partial Differential Equations – Classification of partial differential equations. Finite difference approximations to derivatives, Laplace's equations – Five-point formula, Diagonal five point formula, Jacobi's method, Gauss-Seidal method and successive over relaxation methods. Parablic equation in one space dimension, two levels and three levels difference mthods – Schmidt method, Laasonen method, Crank-Nicolson method, Dufort and Frankel method. Hyperbolic equation in one space dimension, Explicit three level difference scheme, Implicit scheme, First order hyperbolic equation – Lax – Wendroff formula.

Unit 2:

Finite Element Methods – Introduction, Weighted residual methods, Variational methods, Rayleigh-Ritz method, Galerkin method, Finite elements, Assembly of element equation. Application of finite element method, One-dimensional and two-dimensional problems.

Paper – IV : Generalized Hypergeometric Functions and Fractional Calculus

3 hrs. Duration   **Theory Paper**   **Max.Marks-80**
                          Internal Assessment   **Max.Marks-20**
(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).
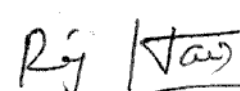
Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1:

Meijer's G-function – Definition, Elementary properties. Multiplication formulas. Derivatives. Recurrence relations. Mellin and Laplace transforms of the G-function.

Unit 2:

H-function of one variable – Definition. Identities. Special cases. Differentiation formulas. Recurrence and contiguous function relations. Finite and infinite series. Fourier series for the H-function. Simple finite and infinite integrals involving the H-function.

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Paper-V: Operations Research-II

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1:

Transportation Models – Mathematical formulation, Initial basic feasible solution, Optimality test, Transportation algorithm for minimization problem, Degeneracy in transportation problems, Unbalanced transportation problem.

Integer Linear Programming – Definition, Gomory's cutting plane method, Branch and Bound method, Applications of Integer programming.

Unit 2:

Dynamic Programming models –Definition, Bellman's principle of optimality, Minimum path problem, Single additive constraint- Multiplicatively and additively separable return, Single multiplicative constraint, Additively separable return, System involving more than one constraint. Applications in production, Inventory control, Linear programming and reliability. Decision Theory.
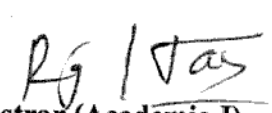
Paper VI: Non-Newtonian Fluid Dynamics

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit 1:

Simple shear, classification of material, Classification of fluid behaviour – Newtonian fluid, Non-Newtonian fluids, Time independent non-Newtonian fluids, Time dependent non-Newtonian fluids, Viscoelastic fluids. Normal stress effects (Weissenberg effect, Merrington effect, reverse circulation etc.). Normal stress material functions. Interpretation of a tangential annular flow experiment and axial annular flow experiment. Extrudate Swell. Shear flow – Kinematics and classification. Kinematics of steady tube flow, Steady tangential annular flow.

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Unit 2:

Unidirectional shear flows. Form of the stress tensor. Steady shear flow material functions. Unidirectional unsteady shear flow material functions. Measurement of viscosity and normal stress coefficients in (i) Cone and plate instrument and (ii) Parallel disk instrument.

Paper-VII: Advanced Graph Theory-II

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.

Unit-I:

Vertex cuts and edge cuts, connectivity and edge-connectivity, bonds, blocks, block-cutpoint graphs, 2-connected graphs, Menger's theorem. Total transformation graphs, Some properties of total transformation graphs.

Unit-II:

Factors, Tutte's 1-factor theorem, Berge-Tutte formula, Petersen's results on 1-factors and 2-factors. Domination Theory - Domination numbers, Some elementary properties.

Paper- VIII: Relativistic Cosmology and Differential Forms

3 hrs. Duration	Theory Paper	Max.Marks-80
	Internal Assessment	Max.Marks-20

(Internal Assessment will be done by Teacher concerned on the basis of test papers, regularity in the class and performance of the Candidate).

Note: This paper is divided into Two Units. Four questions will be set from each Unit. Candidates are required to attempt FOUR questions in all taking two questions from each Unit. All questions carry equal marks.
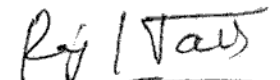
Unit 1:

Non-static cosmological models, Robertson-Walker model and its derivation and Geometrical properties, Fredmann-Robertson-Walker model and its scale factor, Three different forms of scale factor, Doppler effect in Robertson-Walker model, Horizons (Event and Particle), Big Bang Theroy, Steady state theory.

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Unit 2:

Brans-Dicke theory as an alternative theory of gravitation, Derivation of its field equation and solution based on Brans-Dicke Theory. Differential forms: Exterior differentiation, Connection 1-form, Ricci Rotation coefficients, Cartan's equation of structure, Calculation of Riemann Curvature tensor using Differential forms, Curvature 2-form for Vaidya metric.

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